

Robust bi-criteria approach to optimize the composition and properties of alloy steels

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Abstract: The article is dedicated to an approach optimizing a task of statistical modeling of the mechanical properties of products in real production metallurgy process. The approach is designed for the benefit of producers-metallurgists aimed at providing panels of steels of a specific set of eventual industrial properties. This is accomplished by a procedure of composition optimizing based on existing certificates of brands alloy steels. Since the composition of alloys is searched in accordance with the previously announced requirements, it results in a system of limitations, and the task is formulated as a task of decision support. The article presents an approach using mathematical models of optimization problem following the implementation of two approaches (the classical one and Taguchi methodology) capable of decision-making in the production practice.

Key words: Optimizing properties, Statistical modeling, Taguchi methodology, alloy steels.

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1. Introduction

Modern steel industry has to improve the parameters of creating products taking into account environmental and economic constraints. In this aspect competing steel companies need to have software tools and approaches to assist their activity in finding rational decisions related to the impact of composition and processing parameters on the final properties of the manufactured products. The possession of such tools helps to monitor and control production and technology change with reaching various technological properties of the final product. The creation of mathematical models to analyze the objects of metallurgical process under examination is an important stage in achieving this goal. These models contribute to improve

the set of properties and the final product quality. It is confirmed that it is possible to meet the requirements of the current market by implementation of such models. This is the reason for more frequent publication of articles dealing with optimisation aspects of different problems related to metallurgical industry, [1] – [6].

The wide range of problems, which Taguchi method has been applied to, is shown in [17].

The core of Taguchi approach consists of the method for reducing the influence of factors called noise (disturbing) that impair the quality parameters of the product/process. It is where the radical difference from the traditional technique of quality, which provides identification of existing sources and conduction of measurements that are often costly due to their control. The parametric design of Taguchi ensures non-sensitivity to (interference) noise along the way to the proper selection of certain parameters called controllable factors. The centerpiece of this approach is the method of reducing the impact of factors called noise (disturbing) that reduce the product/process quality parameters. It is where the radical difference from the traditional technique of quality, which provides identification of existing sources and conducting measurements often costly due to their control, lies. Taguchi parametric design ensures non-sensitivity to (interference) noise through a proper selection of certain parameters called controllable factors.

The aim of this study is to present a robust approach for determining the influence of alloying elements on the properties of iron-based alloys that ensures better results than the input ones used to obtain a mathematical model.

The formulated optimization models are used at the stage of modeling the mechanical properties of the composition of low-alloy and medium-alloy structural steels during the production metallurgy process. The proposed approach facilitates the optimization of the steel alloy chemical composition by heat treatment, hardening and high temperature tempering improving the properties of the final product. These requirements generally are followed according to the standards but also may be associated with certain additional requirements claimed by users. All these pre-imposed conditions lead to a set of constraints that must be satisfied by acceptable solutions. Some restrictions can be defined as relations with true quantitative nature. This is especially important to restrictions on mechanical properties of the final product. Their proper formula is based on good mathematical models describing the effect of alloy composition and processing parameters on the final properties of rolled sheets and profiles.

However, metallurgical processes are so complex that it is often impossible to be described by formally analytical expressions. The statistical analysis of industrial data is an important and supporting alternative in such cases. That is why we have limited the field of study only to the influence of the chemical composition of the heat-treated alloys on the set of properties. In the context of the analysis of metallurgical processes, different methods to study the data described in the references can be found, [7] – [9].

The multidimensional regression analysis is one of the most popular data-mining methods. It has been applied successfully to the study of multiple relationships in metallurgical industry, [10] – [12]. Mathematical models based on chemical composition have been reported in [15] and [16].

Due to the nature of each statistical analysis, the coefficients of limitations imposed by the regression analysis are known only approximately. That should be reflected in the mathematical model of the optimization problem. The statistical analysis presented in this article is based on of data collected during the real production process described in [[http://www.splav.kharkov.com / choose_ type.php](http://www.splav.kharkov.com/choose_type.php)].

The range of change of the used alloying elements of ferrous alloys are listed in Table 1.

Table 1. Minimum and maximum values of alloying components

Input parameter	Chemical symbol	min [%]	max [%]
x ₁	C	0.12	0.52
x ₂	Si	0.27	1.4
x ₃	Mn	0.35	1.75
x ₄	Ni	0	4.22
x ₅	S/P	0.025	0.035
x ₆	Cr	0.15	2.50
x ₇	Mo	0	1.5
x ₈	V	0	0.45

Regardless of that, the proposed optimization approach for modeling the final mechanical properties of alloys can be applied to any production process with steel manufacturing.

2. General description of the approach

The analysis presented in this paper is related to the analysis of mechanical properties of steel specimens described by the following parameters: yield strength, Re [MPa], tensile strength, Rm [MPa], relative elongation (A [%]), relative shrinkage (Z [%]), impact strength (KCU [kJ/m^2]). The limitations connected with these parameters are due to steel grade characteristics and customer's specifications. However, the main problem is that these parameters cannot be under direct observation during the manufacturing process, so any limitations associated with them can not be clearly defined in the optimization model. That means that we must develop models linking the final mechanical properties of the specimen/sample of the steel chemical composition as well as the parameters of the production process.

The regression analysis allows describing the relation between the variables of input and output, without going into the phenomenon nature during the process.

The regression models presented below have been created based on the data collected during the industrial production process.

The statistical analysis described in this section is based on a data set of 90 records extracted from the whole database.

To estimate the regression coefficients in the final model, different methods from references can be considered in the context of the study on industrial data, [13-14].

The Least Squares method, LS is used to estimate the regression parameters. The estimated models of parameters Rm , Re , and A , Z and KCU obtained in the examinations are given below.

In respect to the problem under examination, nonlinear regression dependencies have been identified for each of the mechanical properties of steels. The regression dependencies are of the following kind:

$$f_i(x) = b_{00}^i + \sum_{j=1}^8 b_{j0}^i x_j + \sum_{j=1}^8 b_{jj}^i x_j^2 + \sum_{j=1}^8 \sum_{l=j+1}^8 b_{jl}^i x_j x_l$$

Here b_{ij} are the regression model parameters. The coefficients in equations are defined in Table 2. The models can be used for prediction if the check-up $F > F(0.5, v_1, v_2)$ described in details has been made.

Table 2. Coefficients of regression models of the examined target parameters.

No	Coefficient	Re [MPa]	Rm [MPa]	A [%]	Z [%]	KCU [kJ/m ²]
1	Free member	2114.115	-130.007702	46.556931	147.9264	-4001.027064
2	X ₁	7756.8407	11862.25089	-5162068	79.41095	6322.9676840
3	X ₂	7199.188	6890.25190	-39.72085	80.01231	372.74973729
4	X ₃	-7695.056	-5461.20300	-11.96989	-74.65328	940.57612619
5	X ₄	-208.3606	182.869253	-6.194367	5.2045198	488.81611523
6	X ₅	-206849.1	-134748.77	608.78794	-6864.399	152954.11975
7	X ₆	3214.6636	2695.42038	-44.13384	-1.933543	2329.2901438
8	X ₇	-25808.88	-21850.549	296.07430	-267.2699	-3072.4165598
9	X ₈	88806.163	73735.9515	-1272.128	357.52104	-3660.1575070
10	X ₁ X ₂	-386.6554	-744.96043	-17.34393	-17.50373	-1445.4379850
11	X ₁ X ₃	-1070.550	-876.13841	4.4991108	12.100853	-1370.0424233
13	X ₁ X ₅	-136396.0	-249603.31	-479.9262	-3507.860	119.87525798
14	X ₁ X ₆	-937.1690	-1007.3471	10.247712	-4.990204	-1884.4471846
15	X ₁ X ₇	-4306.703	-5097.3821	56.210086	20.372882	4201.0341778
16	X ₁ X ₈	28210.222	24310.7901	-234.0791	-22.71132	-1428.2331137
17	X ₂ X ₃	-2441.408	-2127.89081	8.1761576	-14.13188	-30.280655834
18	X ₂ X ₄	1139.5872	724.553778	9.5115582	5.9539844	-282.30199391
19	X ₂ X ₅	-136064.2	-119201.12	630.64368	-344.99676	8526.8578246
20	X ₂ X ₆	-3798.235	-3188.333	24.332090	17.4805982	1055.3547647
21	X ₂ X ₇	90881.356	81067.277	-1331.408	849.686125	4391.3113008
22	X ₂ X ₈	-385523.8	-333160.54	5120.2990	-3249.86576	-13145.188284
23	X ₃ X ₄	461.26479	598.74288	-3.247065	3.139703587	396.96680405
24	X ₃ X ₅	235316.49	182688.012	-444.5946	1720.234197	-29851.692269
25	X ₃ X ₆	-45.97452	-174.831984	11.423925	3.063216433	-263.39343049
26	X ₃ X ₇	3249.6850	3079.279696	-13.73530	-8.65473970	1821.7477377
27	X ₃ X ₈	-4913.023	-5777.88881	110.24208	5.924195776	-13095.534513
28	X ₄ X ₅	-5816.934	-13010.3617	59.110091	-191.099566	-6739.7899380
29	X ₄ X ₆	-159.9173	-183.698860	4.3070441	-2.01496501	-302.50918410
30	X ₄ X ₇	-6.156909	-55.9131990	-2.088778	.3982070443	512.55485652
31	X ₄ X ₈	-105.0768	-101.445659	.73706332	-12.6460954	-1043.2706304
32	X ₅ X ₆	-27149.24	-14448.1896	187.81631	-108.167139	-44266.146045
33	X ₅ X ₇	41678.774	6877.259400	884.54626	1016.286169	-21746.085509
34	X ₅ X ₈	300515.42	384385.0029	-2272.635	9664.029850	278153.48457

35	$X_6 X_7$	-1228.339	-931.670546	32.838349	6.958889962	36.932790319
36	$X_6 X_8$	1056.3228	948.6964392	2.7722696	134.2490637	2222.0903600
37	$X_7 X_8$	3239.2552	3436.360866	51.757562	-129.431204	-6921.624826
38	X_1^2	-992.3896	-1056.09582	2.2481142	64.03036844	-5736.340380
39	X_2^2	2493.7122	1829.847034	-4.790038	-48.9121651	-1038.885976
40	X_3^2	184.24023	-47.9655438	6.8702503	4.221654885	260.99429271
41	X_4^2	34.060169	34.10720898	.22590263	.8361003346	18.759454624
42	X_5^2	3139839.7	2825344.725	-13811.99	116379.3872	-1539971.788
43	X_6^2	-242.5842	-250.198773	3.7158944	-3.00596838	-166.5798330
44	X_7^2	891.86516	573.1606953	-9.448061	-4.11998428	-806.0512447
45	X_8^2	-4756.548	-5883.33206	-356.8126	1181.488066	61291.102691
R		0.876	0.870	0.863	0.837	0.861
F		3.357	3.119	2.99	2.179	2.284
F (0.5, v_1, v_2)		1.655	1.656	1.655	1.684	1.73

The composition optimization is applied only in respect to yield strength Re and respective elongation A. The results are compositions where, using the other models, the other mechanical characteristics can be determined: *Rm*, *Z* and *KCU*.

By Taguchi methodology [18] (Khosrow Dehnad) an experiment modeled on orthogonal matrices developed by him is carried out. The experiment can be accomplished in two ways by:

- a real experiment leading to obtaining results for processing;
- a numerical experiment with the presence of adequate regression models.

The availability of the described model coefficients, which can be used to predict, give a possibility to make a numerical experiment involving Taguchi method. The noise matrix is selected from orthogonal matrix I (27,13) with 27 rows and 13 columns developed by Taguchi and published in [17]. The matrix is worked out with factors at three levels.

Specifically for the data of the experiment, eight of columns are used since the regression models are obtained on the basis of eight variables. In the matrix X_1 corresponds to carbon, X_2 corresponds to silicon, X_3 corresponds to manganese, X_4 corresponds to nickel, X_5 - corresponds to sulfur and phosphorus, X_6 corresponds to chromium, X_7 corresponds to molybdenum and X_8 corresponds to vanadium.

The methodology proposed is implemented for yield strength Re and relative elongation A. To take out the models of these two target functions, 90 measurements that form the data

matrix A (90, 8 +1) have been used. Here the added column "1" is for the output target function Re or A stored compactly in the matrix.

To optimize the computing process, the scheme, which having been processed for the particular case takes the following kind, is selected.

In numerical experiments that use models based on the chemical composition the noise can be expressed only in the change of the respective components. It is assumed to express noise Δ in the following way $\Delta_i = \frac{\bar{x}_i}{k}$ where further calculations are made for k equal to 100 and 70.

Here \bar{x}_i is the mean value of relevant variable "i".

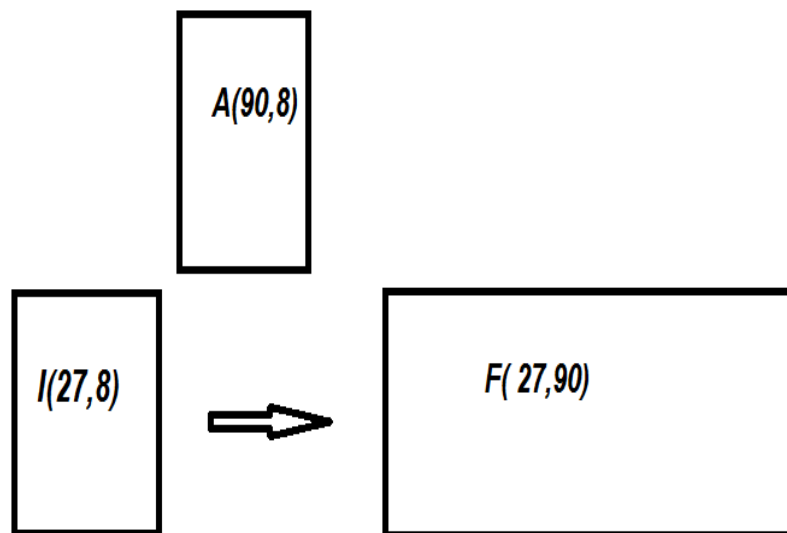


Fig. 1. Organizing experiments with parametric planning with matrices I, A and F

For level "1" of I (27,8) noise is subtracted from relevant x_i taking the value of $x_i - \Delta_i$. With level "2" no correction is applied, the value of x_i is preserved. With level "3" noise is added to relevant x_i taking the value of $x_i + \Delta_i$. In numerical experiments where models based on chemical composition are used, noise can be expressed only in the change of the respective components. Noise Δ is assumed to be expressed as follows $\Delta_i = \frac{\bar{x}_i}{k}$, where the further calculations are made for k equal to 100 and 70. Here \bar{x}_i is the average value of the respective variable "i". In level "1" I (27,8) noise is subtracted from respective x_i taking the value of $x_i - \Delta_i$. In level "2" no correction is applied, the value of x_i is preserved. In level "3" noise is added to respective x_i taking the value of $x_i + \Delta_i$.

Thus, noise is expressed in the change of chemical composition. The calculation process is organized as follows:

A row of matrix I (27,8) is taken (for example, row 1 - I (1,8)). In this row level "1" is assigned for each x_i , i.e. noise will be taken out from each value x_i .

Thus F (1,1) of the matrix F (27,90) is obtained from the first row of A (90,8). The same rule is applied to the rest of the series F (90,8) and it forms F (27,90).

It is continued with the next row of matrix I (27,8) performing the following sequence. Each row of matrix I (27,8) forms a relevant row of matrix F (27,90).

If we take the first column of matrix I (27,8) relevant to X_1 , it is seen that the first nine rows correspond to level "1" of noise, the second nine lines correspond to level "2" and the third nine rows correspond to level "3" of noise. This makes possible to use the values of the first nine rows of matrix F (27,90) to calculate level "1", to use the second nine rows to calculate level of "2" and the third nine rows for calculation at level "3" for X_1 . For other columns from 2 to 8 it is necessary to sort in ascending order X_i from I(27,8). After sorting the column obtains the kind of the first column. If changes are made with sorting, they are reflected also in matrix F (27,90). After sorting of the respective variable, calculations for different levels can be made. It is continued with the next matrix row I (27,8) performing the following sequence. Each row of matrix I (27,8) forms a corresponding row of matrix F (27,90). If we take the first column of matrix I (27,8) corresponding to the X_1 , it is seen that the first nine rows correspond to noise level "1" of noise, the second nine rows correspond to level "2" and the third nine rows correspond to noise level "3". That allows using the values of the first nine rows of matrix F (27,90) to calculate level "1", the second nine rows to calculate level "2" and the third nine rows to calculate level "3" for X_1 . For the rest columns from 2 to 8 it is necessary to sort by ascending order of X_i of I(27,8). After sorting the column takes the kind of the first column. With sorting, if shifts are made, they are reflected in matrix F (27,90). After sorting the corresponding variable it is possible to make calculations for different levels. In the numerical experiment noise was first determined with $K1 = 100$. The analysis of the graphics below shows low sensitivity for both Re and A. For this reason, an experiment with $K2 = 70$ has been made as well. In these calculations, as shown in the attached graphics, the results are sharper for both target functions under examination. As explained above in this section, in compliance with Taguchi, the higher evaluation is taken as the optimal value. It

defines the noise level and the direction of possible further search or further search should be carried out by the component. Fig. 3- Fig. 10 contain Taguchi assessments of Re and A.

Thus, for each level of noise 810 different values of this function are obtained and that ensures reliable outcomes.

With the numerical experiment the noise was set first with $K1 = 100$. The analysis of the graphics below shows low sensitivity for both yield strength Re and A. For this reason an experiment with $K2 = 70$ has also been made. In these calculations, as shown in the attached graphics, the results are sharper for both objective functions under examination. As explained above in this section, according Taguchi the value accepted as optimal is the one where the assessment is greatest. It defines the noise level and the direction of possible further search or most generally the further search should be carried out by the component. Fig. 3 - Fig. 10 contain Taguchi assessments of Re and A. Calculations are performed according to the following algorithm.

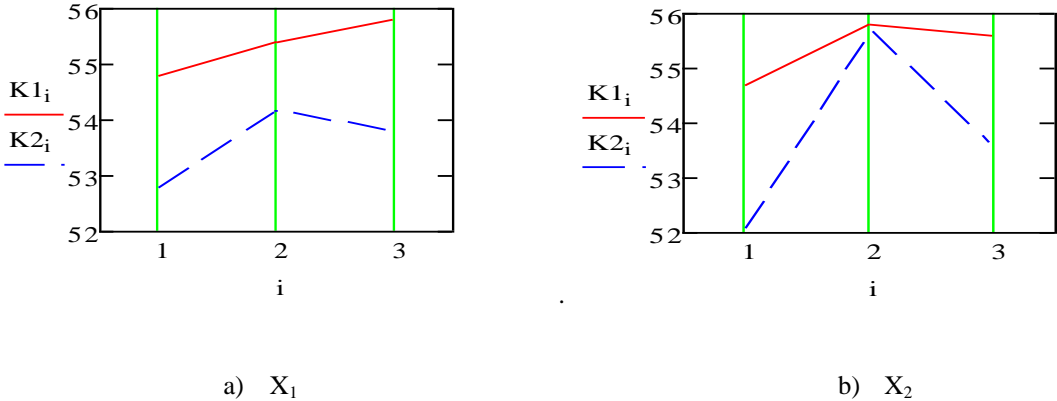


Fig. 3. Taguchi assessments of yield strength Re with examination of: a) carbon; b) silicon.

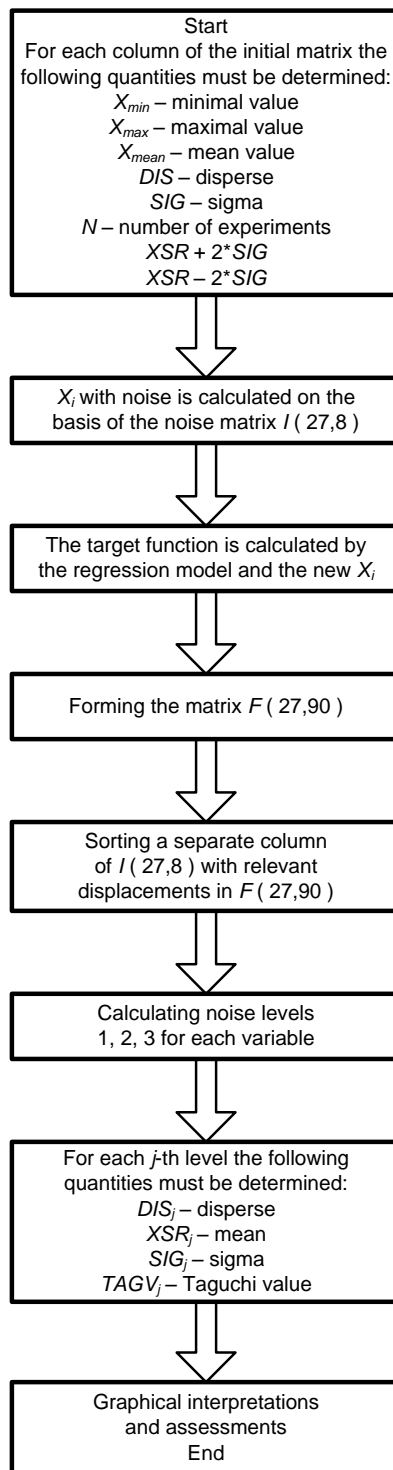
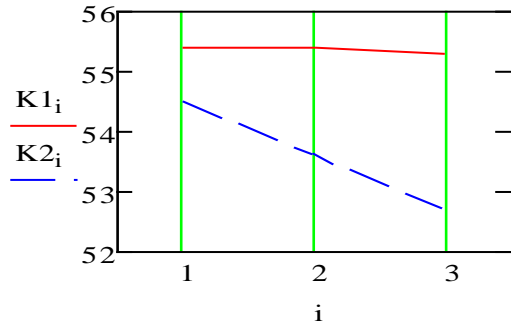
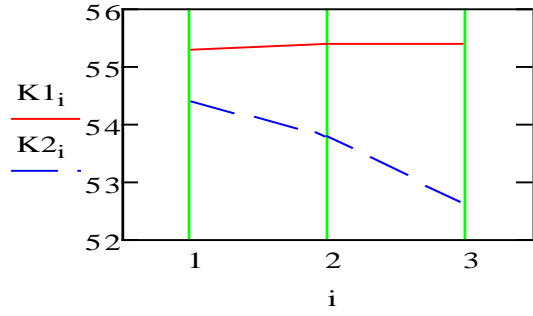


Fig. 2 Computational algorithm

Thus 810 different values of this function are obtained for each level of noise and this ensures reliable outcomes.

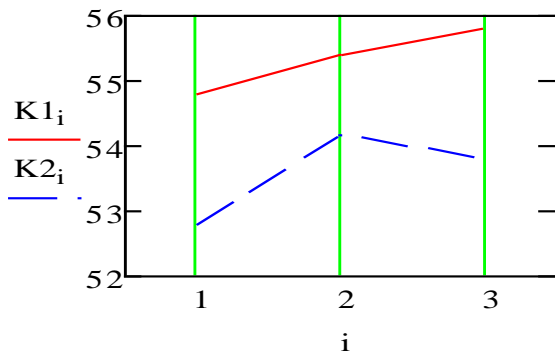


a) X₃

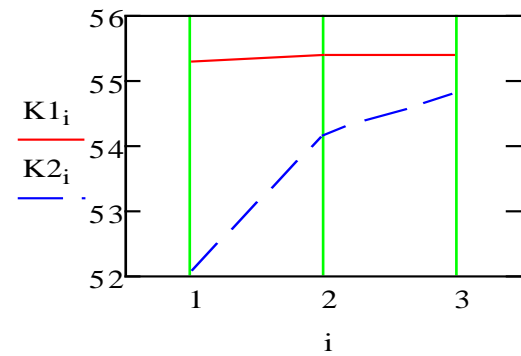


b) X₄

Fig. 4. Taguchi assessments of yield strength Re with examination of: a) manganese; b) nickel.

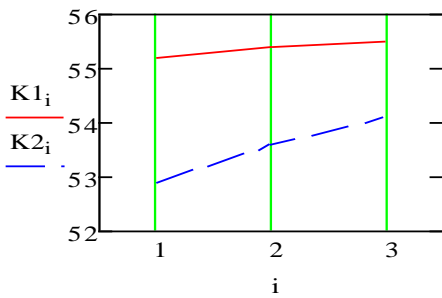


a) X₅

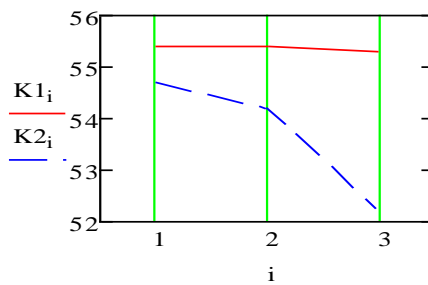


b) X₆

Fig. 5. Taguchi assessments of yield strength Re with examination of: a) sulfur; b) chromium.

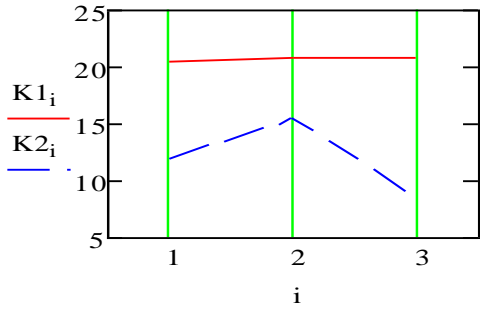


a) X₇

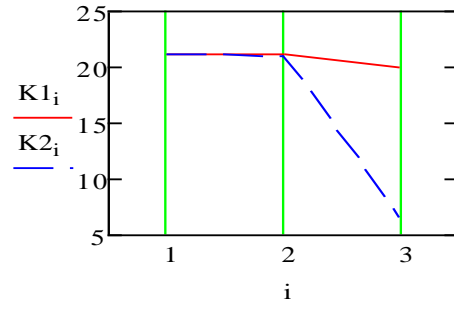


b) X₈

Fig. 6. Taguchi assessments of yield strength Re with examination of: a) molybdenum; b) wolfram.

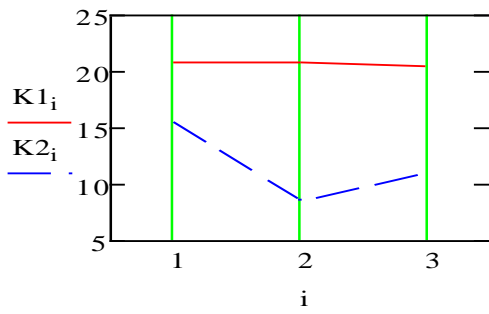


a) X₁

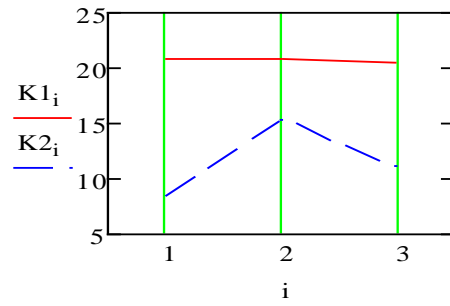


b) X₂

Fig. 7. Taguchi assessments of relative elongation with examination of: a) carbon; b) silicon.

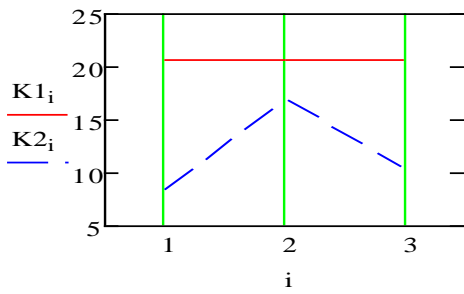


a) X₃

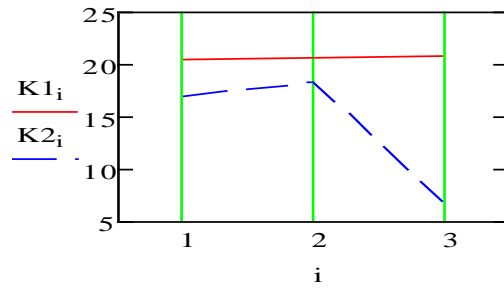


b) X₄

Fig. 8. Taguchi assessments of relative elongation with examination of: a) nickel; b) manganese.



a) X₅



b) X₆

Fig. 9. Taguchi assessments of relative elongation with examination of: a) sulfur; b) chromium.

The results of the relative elongation A are more interesting. The four variables, carbon, nickel, sulfur and chromium, should not be changed from their basic levels, and the rest variables, silicon, manganese, molybdenum and vanadium are expected to change in direction to decrease of their values.

3. Search of optimal composition

As the experiment is numerical, it is possible to perform numerical optimization with the mathematical models obtained as the values of X_i are remained to change within the limits defined by the output data (Table 1).

Different methods for numerical optimization are described in [19] and [20]. The simplex method of Nelder and Mead with a deformable polyhedron has been selected for the purpose. This method has been chosen because it is a method of direct searching extremes and is suitable for the case of ravine surface of the target function. It has been proven to be one of the most effective methods, especially when the number n of variables is $X_i \leq 6$. Particularly, in the yield strength $Re - X_2$ and X_5 the variables are not changed and therefore the number remains six variables. With A those that do not change are $X_1 X_4 X_5$ and X_6 . Respectively for A the number variables that remain is 4 (according to Table 4).

That some of the variables remain unchanged, i.e. keep their initial values imposes the necessity to separately carry out optimization for the chemical composition of each steel. The above mentioned X_i values are kept at their initial levels and optimization is carried out by modifying the others. The change of X_i is in the range of the output data set in Table 1. Thus ninety different optimizations with ninety different chemical compositions are carried out as for each case a single value of extreme – maximum is obtained.

Then all maxima are sorted in ascending order and those that satisfy the pledged desire to be larger than the largest ones in the output data are selected. Specifically for elongation $A = 26\%$, yield strength $Re = 1375$ MPa. Such an approach is justified because if the problem is considered from the viewpoint of technology, the individual optimization is an improvement of individual, actually existing steel that has proved to belong to a certain class. It is easier to improve something existing rather than create a new one. If the problem is examined from the viewpoint of the optimization, there is a case of searching the extreme of many starting points, something recommended in searching a global extreme.

When processing the results by the way already cited for relative elongation A, values obtained exceeded the value of A = 26% such as A = 27.89%, A = 27.80%, 27.01% = A and A = 26.90%. It is a proof that this indicator can be modified in the desired direction upwards.

The results for yield strength Re are more interesting. Of all cases after optimization Re accepted 16 different values. The greatest number (40 cases) is for Re = 1132 [MPa] – (1) followed by Re = 2058 [MPa] – (2), (25 cases), although in these 65 cases there are different initial values of unchanging X_i. The results of this analysis are shown in Fig. 11.

This once again supports the requirement to look for the extreme of multiple starting points and the approach applied.

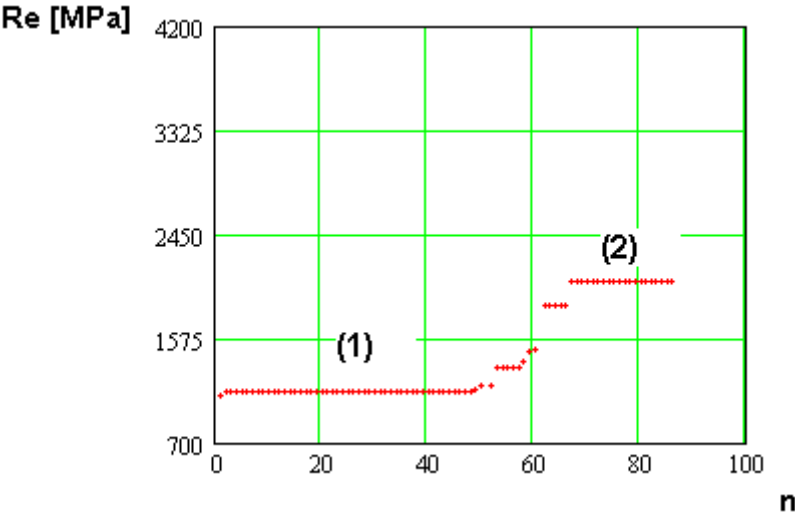


Fig.11. Sorting yield strength Re values in ascending order.

The choice of alternatives for elongation A is smaller and therefore the respective value of the yield strength limit is determined for them (with the corresponding chemical composition) thus determining the final decision. Only for one of the cases the pre-set requirement to improve yield strength Re has been fulfilled.

A [%] calculated	C	Si	Mn	Ni	S and P	Cr	Mo	V	Re [MPa] calculated
27.89	0.355	1.25	1.75	0.3	0.025	1.25	0	0	2989

To improve quality in terms of both examined characteristics, yield strength R_e and relative elongation A in existing steel 35HGSA, which is GOST regulated, it is recommended to increase the amount of manganese from 1.1% to 1.75%. This outcome indicates that the task is feasible and the approach applied can result in improvement of the alloy composition. That can be done with a real experiment where better characteristics are obtained after correction related to manganese from the quantity defined in the certificate of 35HGSA.

Conclusion

The numerical experiment has proved the ability to improve the quality of steel of a certain class. Mathematical models suitable for forecasting and optimization have been derived. The approach of Taguchi applied has led to a desired result, to separate variables X_i for the examined parameters that do not influence significantly on the final result. With this limit, the numerical optimization for maximum search has been conducted with each chemical composition. That allows improving it. Relative elongation A turned to be less variable index and yield strength R_e requires caution with extreme selecting. The decision of bi-criteria problem set has been defined thus proving that the Taguchi approach is applicable to a similar class of problems.

Reference:

1. Paiva A.P., E.J. Paiva, J.R. Ferreira, P.P. Balestrassi, S.C. Costa, A multivariate mean square error optimization of AISI 52100 hardened steel turning, *International Journal of Advanced Manufacturing Technology* **43**, 631-643 (2009).
2. Ray P.K., R.I. Ganguly, A.K. Panda, Optimization of mechanical properties of an HSLA-100 steel through control of heat treatment variables, *Materials Science and Engineering* **A346**, 122-131 (2003).
3. Dulikravich G.S., I.N. Egorov, Robust optimization of concentrations of alloying elements in steel for maximum temperature, strength, time-to-rupture and minimum cost and weight, *Conference on Computational Methods for Coupled Problems in Science and Engineering – Coupled Problems 2005*,
4. Liu Xiang Hua, Lan Hui Fang, DuLin Xiu, LiuWeiJie, High performance low cost steels with ultrafine grained and multi-phased microstructure, *Science China Technological Sciences* **52**, 8, 2245-2254 (2009).
5. Kusiak J., A. Zmudzki, A. Danielewska - Tuleca, Optimization of material processing using a hybrid technique based on artificial neural networks, *Archives of Metallurgy and Materials* **3**, 50, 609-620 (2005).
6. Sidorina T.N., I.V. Kabanov, Optimization of carburizing steels for drilling tools within grade chemical composition, *Metal Science and Heat Treatment* **49**, 9-10 (2007).

7. Moravka J., K. Michalek, B. Chmiel, Statistical analysis of heats with targeted overheating realized in the EAF at Trinec Steelworks, Archives of Metallurgy and Materials **2**, 53, 1-8 (2008).
8. Xie H.B., Z.Y. Jiang, X.H. Liu, G.D. Wang, A.K. Tieu, Prediction of coiling temperature on run-out table of hot strip mill using data mining, Journal of Materials Processing Technology **177**, 121-125 (2006).
9. Talar J., Data mining methods – application in metallurgy, Archives of Metallurgy and Materials **2**, 52, 239-250 (2007).
10. Potemkin V.K., O.S. Khlybov, V.A. Peshkov, Complex mathematical model for predicting mechanical properties and structure of steel sheets, Metal Science and Heat Treatment **42**, 11-12 (2000).
11. Palanisamy P., I. Rajendran, S. Shanmugasundaram, Prediction of tool wear using regression and ANN models in end-milling operation, International Journal of Advanced Manufacturing Technology **37**, 29-41 (2008).
12. Rodrigues P.C.M., E.V. Pereloma, D.B. Santos, Mechanical properties of an HSLA bainitic steel subjected to controlled rolling with accelerated cooling, Materials Science and Engineering **A283**, 136-143 (2000).
13. Dima I.C., A.Z. Grzybowski, J.K. Grabara, G.R. Goldbach, O.R. Popescu, Utilizing of mathematics-statistics methods concerning mechanic properties of heavy steel plates – dealing with the ill conditioned data – [in:] Proceedings of International Conference on Mathematical Models for Engineering Science (MMES '10) (ed. V. Mladenov, K. Psarris, N. Mastorakis, A. Caballero, G. Vachtsevanos), WSEAS Press, 277-284 (2010).
14. Grzybowski A.Z., Z. Urbanowicz, Alternative methods of regression in modeling properties of steel plate – a comparative studies, [in:] Proceedings of 3d International Conference on Parallel Processing & Allied Mathematics, Kazimierz Dolny, 533-542 (1999).
15. Efimychev, Y., Mikhailov, S., Svyatkina, B., & Prokhorov, I. (1976). Regresiony analysis of quality steels and alloys. Moskow: Metallurgy.
16. Rozhkov, I., Vlasov, S., & Mulk, G. (1990). Mathematical models for the choice of optimal technology and quality control of steel. Moskow: Metallurgy.
17. Tontchev N., Y. Kalev Determining influence of alloying elements on properties of alloys by robust experiment, MEST Journal, Vol.1, №1, 2013, http://mest.meste.org/MEST_Najava/II_tontchev.pdf
18. Khosrow Dehnad, Quality control., Robust design and the Taguchi method, AT&T, 1989.
19. Bunday B.D., Basic optimization methods, Edward Arnold, 1984.
20. Vuchkov, I., & Stoyanov, S. (1980). Mathematical modeling and optimization of technological objects. Sofia: Technique.

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