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CHAOTIC BEHAVIOR OF MECHANICAL VIBRO IMPACT SYSTEM WITH TWO DEGREES OF FREEDOM AND POSSIBILITIES OF CHAOTIC BEHAVIOR OF QUARTER VEHICLE MODEL

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Abstract

In this paper, a mechanical vibro impact system with two degrees of freedom is analyzed. The dynamical behavior of the system is determined using different tools from dynamical systems theory, including bifurcation diagrams, largest Lyapunov exponent, and Poincare cross sections. Depending on the angular velocity of the excitation, the system shows both periodic and chaotic behavior.The same procedure is than used to examine the possibility of chaotic behavior of quarter vehicle model.

Key words: mechanical vibro impact system, bifurcation diagram, largest Ljapunow exponents, attractors, Poincare sections.

Mechanical vibro impact system with with two degrees of freedom

Mechanical oscillator systems, in which impact loads occur because of mutual collision of its parts or because of collision with rigid obstacles, are called impact oscillators or vibro impact systems [13], [14]. A Vibro impact phenomenon, itself, is present in different areas of applied mechanics and engineering, and often is unwanted. Its presence leads to weakening of the system, noise, even to system destruction.

Regardless the fact, weather the impact loads in the system is pleasant or not, research of such systems makes certain interest. Knowing that the differential equations of motion of a vibro impact system are with disturbed continuity, it is more convenient to observe the systems behavior with the chaos theory tools, than to do it directly through the differential equations of motion.

$$F_1=a_1 sin(\omega t) \qquad F_2=a_2 sin(\omega t)$$

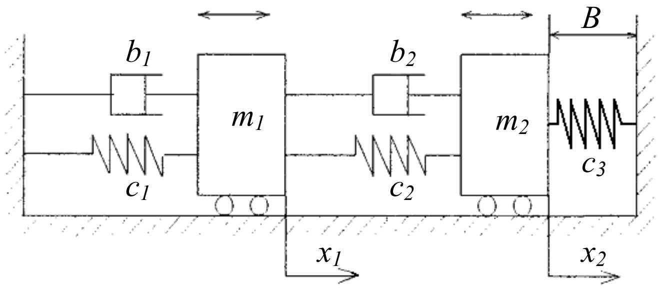

Fig.1. Model of vibro impact system with two degrees of freedom

A model of vibro impact system with two degrees of freedom is shown on fig.1 [14]. The system oscillates under the extern excitations F_1 and F_2. During the oscillations, the mass m_2 impacts the rigid obstacle and so the continuity of the movement of the system changes. Depending on the parameters of the system, the distortion of the continuity of the movement of the system can lead to destruction of the periodical movement and appearance of chaos [8], [9].

The system equations of motion are given below. The impact load is modeled with additional spring c_3, with variable stiffness depending on the position of the mass m_2. In other words, when there is no impact, the stiffness of the spring c_3 is zero, and at the same moment when the impact occurs, the stiffness of the spring c_3 is enormously big and simulates rigid obstacle.

$$m_1\ddot{x}_1 + c_1x_1 + c_2(x_1 - x_2) + b_1\dot{x}_1 + b_2(\dot{x}_1 - \dot{x}_2) = a_1 \sin(\omega t)$$
$$m_2\ddot{x}_2 + c_3x_2 - c_2(x_1 - x_2) - b_2(\dot{x}_1 - \dot{x}_2) = a_2 \sin(\omega t)$$

That is:

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = (a_1 \sin(\omega t) - (c_1y_1 + c_2(y_1 - y_3) + b_1y_2 + b_2(y_2 - y_4)))/m_1$$
$$\dot{y}_3 = y_4$$
$$\dot{y}_4 = (a_2 \sin(\omega t) - c_3y_3 + c_2(y_1 - y_3) + b_2(y_2 - y_4))/m_2$$

Where: $y_1 = x_1$; $y_2 = \dot{x}_1$; $y_3 = x_2$; $y_4 = \dot{x}_2$.

Analyses of the dynamical behavior of the system

Vibro impact system with the following parameters is being analyzed:

$$m_1 = 100\,kg;\quad m_2 = 50\,kg;\quad c_1 = 24213\,N/m;\quad c_2 = 3969\,N/m;$$
$$c_3 = \begin{cases} 0, & (x_2 \le B) \\ 1000\cdot c_1, & (x_2 > B) \end{cases};\quad b_1 = 4000\,Ns/m;$$
$$b_2 = 0;\quad a_1 = 0;\quad a_2 = 100\,N;\quad B = 0.03\,m.$$

The system equations of motion show that its attractor lies in a four dimensional phase space [12], [15]. A bifurcation diagram of the system is shown on fig.2. It shows the relation between the mass m_2 movement and angular velocity of the excitation ω, which is taken as a bifurcation parameter.

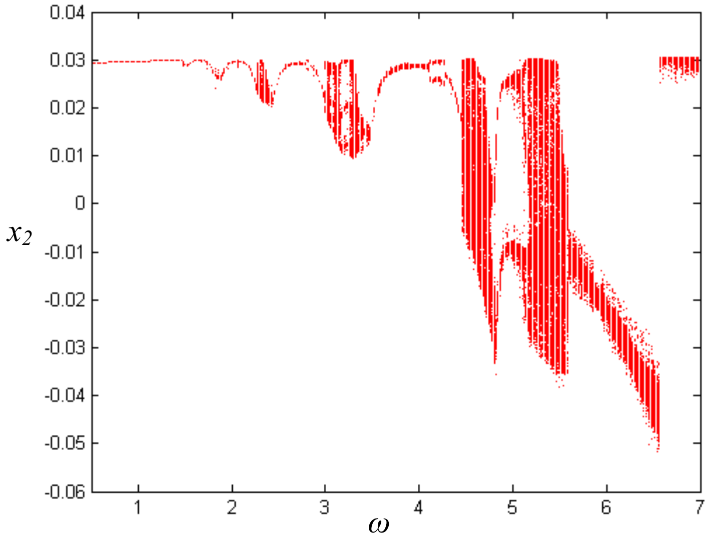

Fig.2. Bifurcation diagram

The projections of the Poincare sections on the x_2 axis, for different values of the angular velocity ω, are given on the ordinate axis. The bifurcation parameter ω which is in between *0,5* and *7 rad/s*, with step of *0,02 rad/s*, is given on the abscissa axis.

The bifurcation diagram implies on a variable nature of the system behavior, depending on the angular velocity of the excitation. At small values of the angular velocity, the system behavior is periodical. On the other hand, at greater values of the angular velocity, the system behavior is chaotic. Additional proof for the chaotic behavior of the system is the positive value of the largest Ljapunow exponent for certain values of the angular velocity [8], [9]. The variation of the largest Ljapunow exponent in relation with the angular velocity of the excitation, which refers to the bifurcation diagram from fig.2, is shown on fig.3.

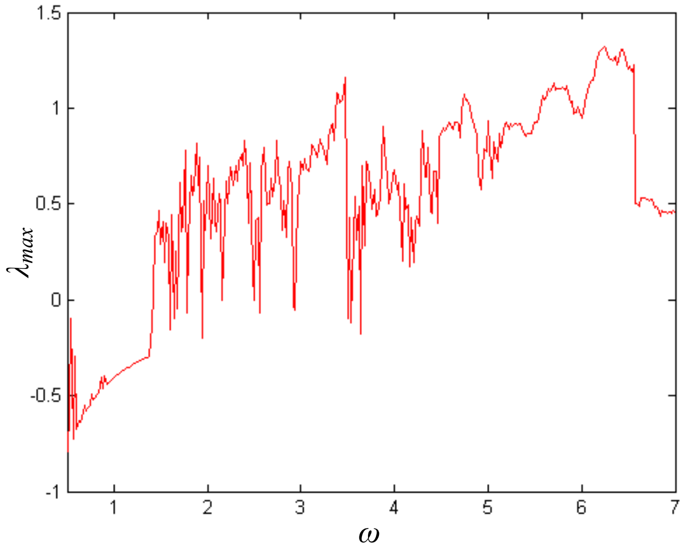

Fig.3. Largest Ljapunow exponent λ_{max} in relation with the angular velocity of the excitation ω

Combined analyses of the bifurcation diagram and the variation of the largest Ljapunow exponent λ_{max} in relation with the angular velocity of the excitation ω, exactly defines the zones of periodic and chaotic behavior of the system. System attractors and Poincare projections for different values of ω are shown on the following figures.

For the ω interval *(0÷1.41) rad/s*, the system behavior is periodic. The projection of the periodical system attractor on $(x_2, \dot{x}_2)$ plane and projection of the Poincare cross section on the same plane, for ω*=1.3 rad/s*, are shown on fig.4. It can be noticed, from the figure, that the motion of the system is with the same period as the excitation of the system.

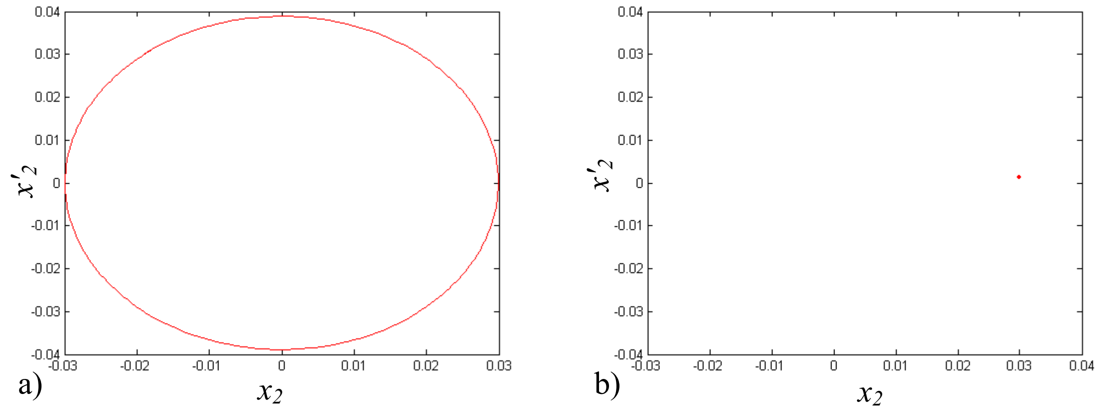

Fig.4. Projection of the system attractor (a) and Poincare cross section (b) for *ω=1.3 rad/s*

All Poincare projections, in this part, are obtained at the moments when the excitation reaches its maximum value. Crossover from periodic to chaotic behavior happens in ω interval *(1.41÷1.43) rad/s*. With bifurcation parameter increasing, the system wonders between chaotic and periodic behavior. This wondering lasts till ω*=3.644 rad/s*.

A projection of a periodic system attractor and Poincare cross section from the zone with dominant chaotic system behavior, for ω*=3.64 rad/s*, are shown on fig.5.

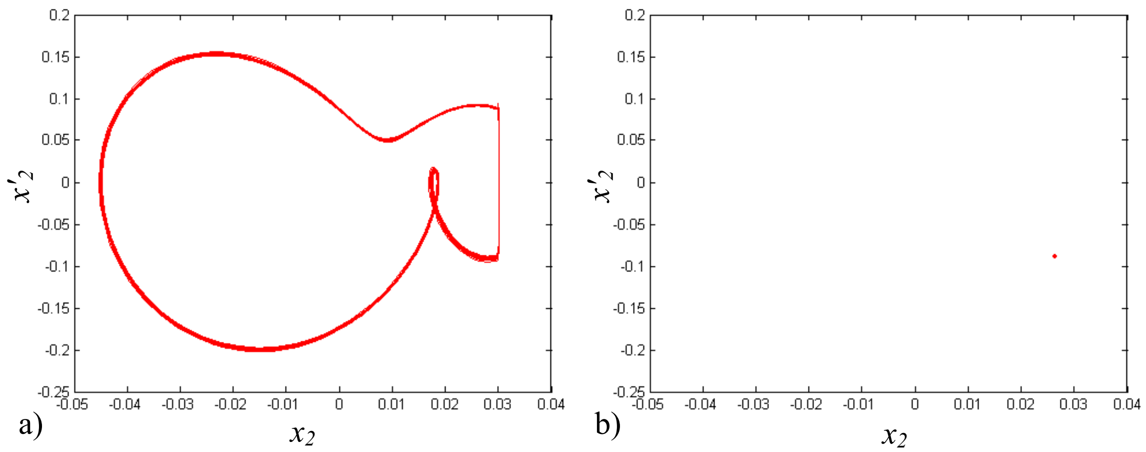

Fig.5. Projection of the system attractor (a) and Poincare cross section (b) for
ω=3.64 rad/s

Projections of chaotic system attractors and Poincare cross sections for different ω values within the mentioned zone with dominant chaotic system behavior are shown on fig.6. The largest Ljapunow exponents are given within the shown projections.

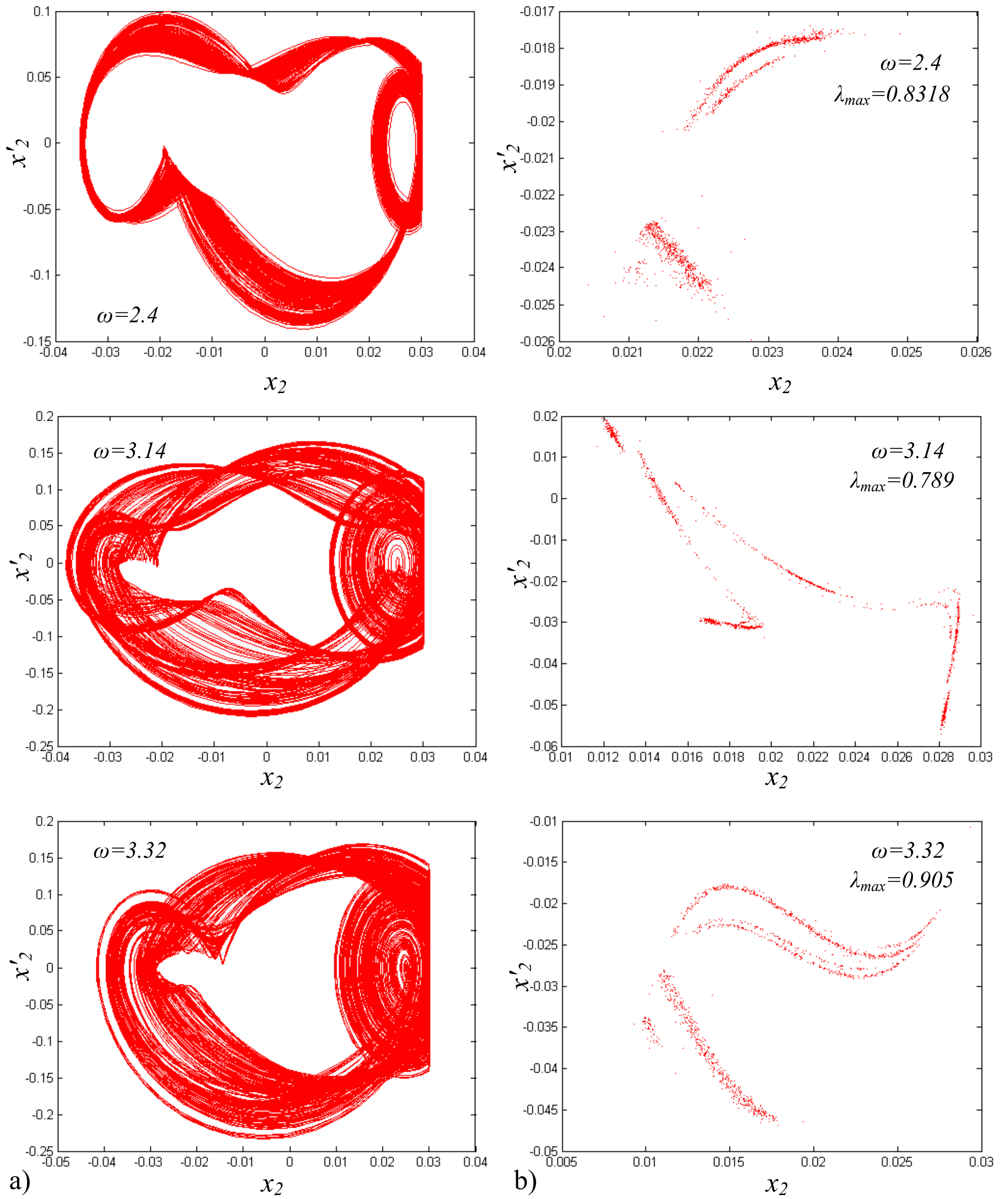

Fig.6. Projection of the system attractor (a) and Poincare cross section (b) for ***ω=2.4, ω=3.14*** **and** ***ω=3.32 rad/s***

With bifurcation parameter increasing over $\omega=3.644$ *rad/s*, the behavior of the system changes from periodic to chaotic and stays chaotic till the end of the observed ω interval.

Projections of chaotic system attractors and Poincare cross sections for different ω values within the zone defined with $\omega>3.644$ *rad/s,* are shown on fig.7.

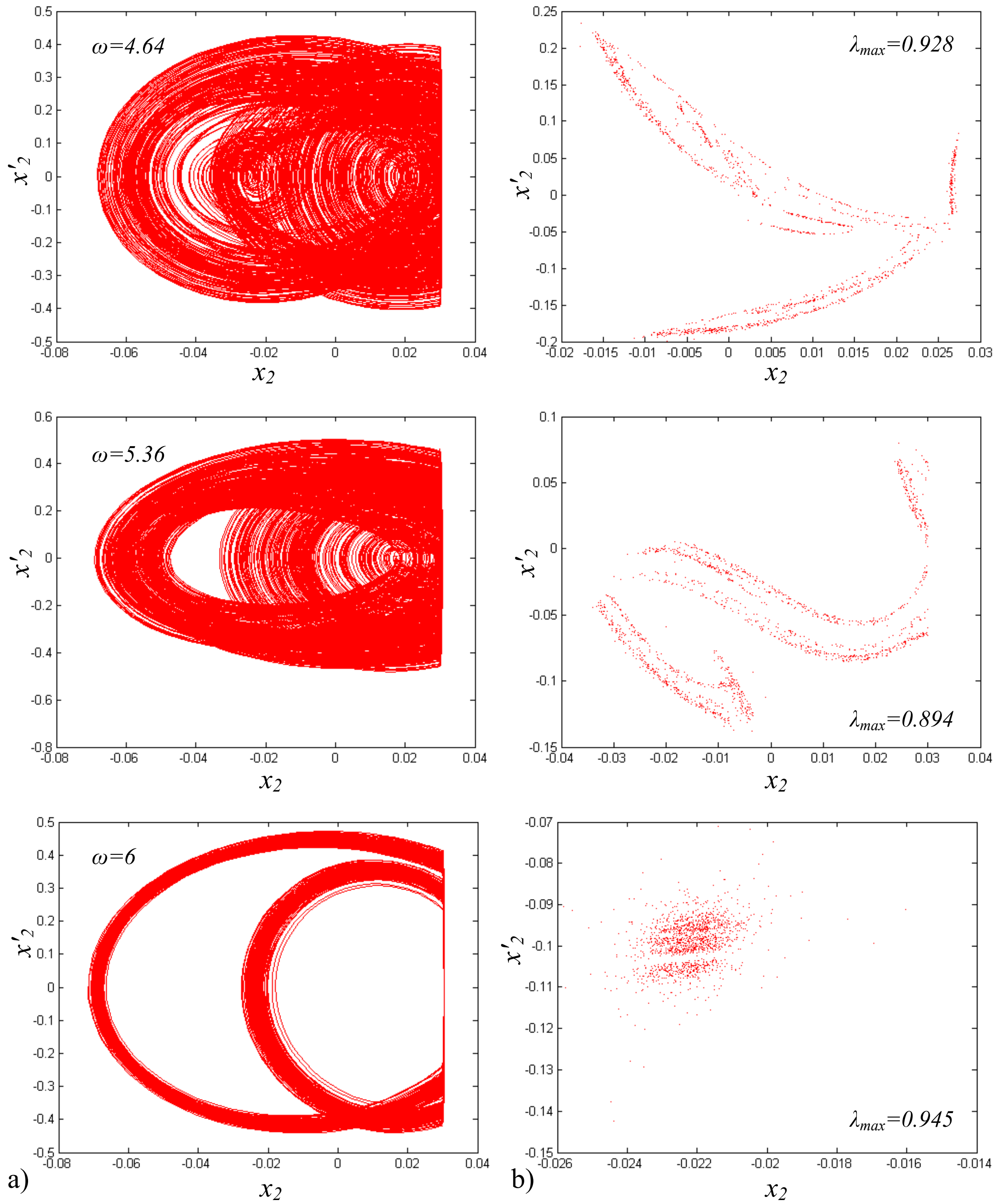

Fig.7. Projection of the system attractor (a) and Poincare cross section (b) for ***ω=4.64, ω=5.36*** **and** ***ω=6 rad/s***

Quarter nonlinear vehicle dynamical model with two degrees of freedom

Quarter vehicle dynamical model with real suspension characteristics is a nonlinear and dissipative system. Considering this fact, the analyses of its dynamical behavior can be conducted with the chaos theory tools.

Because we are considering a mechanical system, with stable linear version, at first it seems that the chaos theory does not overtakes the classical vehicle dynamical behavior research methods. On the other hand, the chaos theory will at least show the same results as the classical methods, and will point out all hidden phenomena in the system behavior, if they exist.

Quarter nonlinear vehicle dynamical model with two degrees of freedom is shown on fig.8 [1], [2], [6]. The equations of motion are given below [4].

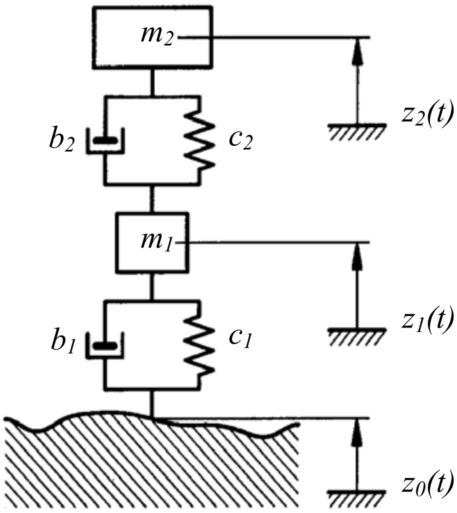

Fig.8. Quarter nonlinear vehicle dynamical model with two degrees of freedom

$$m_1\ddot{z}_1 + c_1(z_1 - z_0) + c_2(z_1 - z_2) + b_2(\dot{z}_1 - \dot{z}_2) = 0$$
$$m_2\ddot{z}_2 + c_2(z_2 - z_1) + b_2(\dot{z}_2 - \dot{z}_1) = 0$$

That is:

$$\dot{y}_1 = -(c_1(y_2 - a\sin(\omega t)) + c_2(y_2 - y_4) + b_2(y_1 - y_3))/m_1$$
$$\dot{y}_2 = y_1$$
$$\dot{y}_3 = -(c_2(y_4 - y_2) + b_2(y_3 - y_1))/m_2$$
$$\dot{y}_4 = y_3$$

Where: $y_1 = \dot{z}_1$; $y_2 = z_1$; $y_3 = \dot{z}_2$; $y_4 = z_2$.

The excitation of the system $z_0(t)$ is introduced through harmonic sine function that simulates the road surface. The velocity of the vehicles model is implicitly presented by the angular velocity of the excitation [3]. In the equations of motion, the dumping in the tyre because of its insignificant value is omitted [4], [5].

Analyses of the dynamical behavior of the system

Quarter nonlinear vehicle dynamical model with two degrees of freedom with the following parameters is being analyzed.

Unsprung mass $m_1=28$ *kg;*

- Sprunged mass $m_2=280$ *kg;*
- Tyre stiffness coefficient $c_1=170000$ *N/m [5];*
- Suspension spring stiffness coefficient $c_2=24123,21$ *N/m;*
- Dumping coefficient at rebound $b_i=2300$ *Ns/m*, dumping coefficient at compression $b_z=1351$ *Ns/m [7].*

The system equations of motion, like those of the above observed vibro impact system, show that the system attractor lays in a four dimensional phase space. The projection of the systems attractor on $(z_2, \dot{z}_2)$ plane, for the velocity of the vehicles model $v=20$ *m/s*, is shown on fig.9.

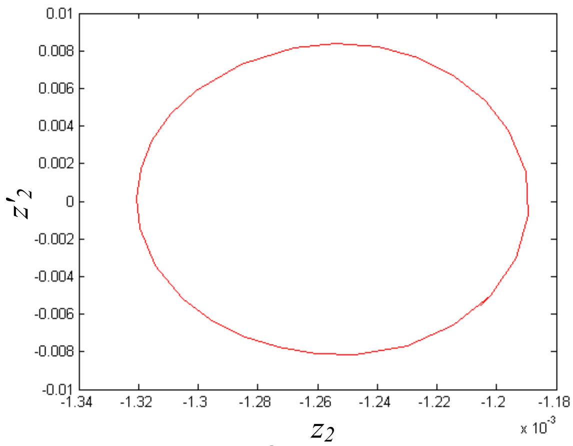

Fig.9. Projection of the systems attractor

It is an ideal attractor that points on a periodic movement of the system, with the same period as the system excitation [8].

The relationship between the movement of the sprunged mass and the velocity of the system, at the moments when the excitation reaches its maximum value, is shown on fig.10.

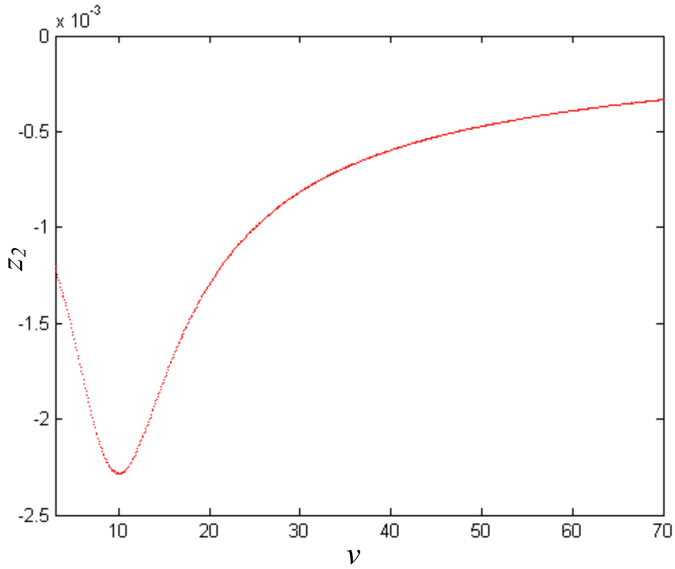

Fig.10. Movement of the sprunged mass in relation with the velocity of the system

The diagram on fig.10 is obtained from the Poincare projections for various system velocities, and indicates on stable periodic behavior of the system through out the whole considered velocity interval.

CONCLUSIONS

The dynamical analyses of the mechanical vibro impact system with two degrees of freedom through the bifurcation diagram, Ljapunow exponents, system attractors and Poincare projections implies on its periodic and chaotic behavior in relation with the angular velocity of the excitation. If the influence of the other system parameters on its behavior is taken into consideration during the dynamic analyses, it will lead to full picture of the conditions at which a chaos exists in vibro impact systems with two degrees of freedom. In this way, with appropriate choice of the system parameters during the design process, the impact load will be set on a level at which it will not lead to an unexpected chaotic system behavior.

On the other hand, the system attractor and the Poincare projections throw out the possibility of whatever chaotic behavior of the quarter nonlinear vehicle dynamical model with two degrees of freedom. This statement opposes some articles that states chaotic behavior of quarter nonlinear vehicle dynamical model with two degrees of freedom [10],[11].

Besides this fact, the vehicle alone is too complex system, just not to consider the idea of chaos elements existing in its behavior. Considering the fact that the equations of motion of a quarter nonlinear vehicle dynamical model with two degrees of freedom are similar to those of the above considered vibro impact system, than there is a great possibility for a chaotic behavior to occur if an impact load is implemented in the system.

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